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DO SOLAR FLARE PROTON EVENTS MEASURE THE GEOMETRY OF A DIFFUSION PROCESS?

ABSTRACT

The temporal evolution of the intensity for solar flare protons, during the onset phase, is shown to be insensitive to the dimensionality of an associated diffusion process.

DO SOLAR FLARE PROTON EVENTS MEASURE THE GEOMETRY OF A DIFFUSION PROCESS?

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I - INTRODUCTION

The temporal distribution of protons arriving at Earth from solar flare events is consistent with an isotropic three-dimensional diffusion process[1,2]. The mean free path (λ) associated with this diffusion process is approximately 0.04 astronomical units[1]. Parker's interpretation[3] of the magnetic measurements of Ness et. al.[4] indicates that there are sharp kinks of the interplanetary magnetic field at intervals comparable to λ. Therefore, a gross picture that emerges is one in which the solar protons that are injected into the interplanetary medium, diffuse to Earth via scattering from "frozen" magnetic kinks[3] which are uniformly distributed over a homogeneous isotropic solar environment. Yet, we already know that the solar corona is not always spherical[5], and the rotating sun spins the magnetic field of the solar wind into an Archimedes spiral[6,7]. We shall show explicitly that the temporal development of the observed solar flare proton intensity is, in fact, a poor indication of the geometry associated with the diffusion process which has been invoked[1,2,3] for describing the propagation of the beam.

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II - THE DIFFUSION PROCESS

The distribution function (ρ) for a particle that is diffusing among a passive lattice of scattering centers obeys a Smoluchowski generalized diffusion equation(8,9) of the form

$$\frac{\partial\rho}{dt} + \vec{\nabla} \cdot \left[\rho \vec{U} + \vec{J}\right] = 0 \tag{1}$$

where $\vec{U}$ is the drift velocity of the lattice (e.g. the velocity of the solar wind) and

$$\vec{J} \equiv -\mathbb{D} \cdot \vec{\nabla} \rho \tag{2}$$

where $\mathbb{D}$ is a diffusion dyadic consisting of nine elements D_{ij}, for (i) and (j) indices that intermix the three spatial dimensions.

Ideally, we shall consider the problem in the reference system of the lattice, or equivalently, we make the approximation that the wind speed is negligible compared with the speed of the solar flare protons. In practice, the velocity of the diffusing proton ($\approx 10^8$ m/sec.) is about three orders of magnitude greater than the wind velocity. Hénce, we take

$$\vec{U} \approx 0. \tag{3}$$

We consider two extreme cases: (i) three-dimensional isotropic diffusion characterized by

$$D_{ij} \equiv (D_3)\ \delta_{ij} \tag{4}$$

when δ_{ij} is the Kronecker delta, and (ii) one - dimensional diffusion represented by

$$\left\{ \begin{array}{ll} D_{ij} \equiv 0 & (i,j) \neq (1,1) \\ D_{11} \equiv D_1 & \end{array} \right\}. \tag{5}$$

At zero time, the particles are injected at the origin of an unbounded medium. For the physical situation of an absorbing boundary at $|\vec{r}| = R$, the description in terms of an unbounded medium remains approximately valid[10] provided that observations are restricted in space ($\vec{r}$) and time (t) to

$$\left\{ \begin{array}{l} |\vec{r}| < R \\ t \leq 10^{-1} \; (R^2/D) \end{array} \right\}. \tag{6}$$

Subject to the specified restrictions (3,6), the solutions to the diffusion equation (1,2) are the associated Green's functions. For three-dimensional isotropic diffusion (4) this is

$$\rho_3 = (4\pi D_3 t)^{-3/2} \; \exp\left[- r^2/(4D_3 t)\right]. \tag{7}$$

For one - dimensional diffusion (5), the solution is

$$\rho_1 = (4\pi D_1 t)^{-1/2} \; \exp\left[- r^2/(4D_1 t)\right]. \tag{8}$$

These solutions (7,8) are similar in form to the corresponding solutions for the problem of the random walk[11] of a particle of speed v executing steps of length L. For the three-dimensional case, we have

$$\rho_3 = \left(\frac{4}{3} \pi L_3 vt\right)^{-3/2} \exp\left[- r^2/\left(\frac{4}{3} L_3 vt\right)\right]. \tag{9}$$

For one-dimensional random walk, the solution is:

$$\rho_1 = (4\pi L_1 vt)^{-1/2} \; \exp\left[- r^2/ \; (4L_1 vt) \; \right]. \tag{10}$$

Comparing (9) with (7) and (10) with (8), we make the identifications:

$$D_3 = \frac{1}{3} v \; L_3 \tag{11}$$

$$D_1 = vL_1 \; . \tag{12}$$

For a given observation point (r), we observe from (9) and (10) that the maximum intensity (i.e. peak ρ) occurs at a time (t_m) given by:

$$t_m = r^2/(2vL). \tag{13}$$

By inserting (11) and (12) into (13), this time (t_m) may be re-written in a form that better distinguishes the two extreme cases. For isotropic diffusion (11) we obtain

$$t_m = \frac{1}{6}\left(\frac{r}{R}\right)^2 (R^2/D_3). \tag{14}$$

For one-dimensional diffusion (12)

$$t_m = \frac{1}{2}\left(\frac{r}{R}\right)^2 (R^2/D_1). \tag{15}$$

Hence, for the situation where $D_1 \approx D_3$, the one-dimensional peak intensity reaches the observer after a time interval that is three times longer than the interval required for the arrival of the peak of the isotropic intensity. Moreover, particle escape at the boundary could become important <u>before</u> the idealized one-dimensional peak intensity can be achieved but <u>after</u> the idealized isotropic peak intensity has already been observed. According to the indicated restrictions (6) upon the idealized solutions, this situation would obtain for the case that

$$(r/R)^2 \geq 1/5. \tag{16}$$

Under such circumstances (16), our observations near Earth would be rendered relatively insensitive to the one-dimensional component of a diffusion process.

III - A GRAPHICAL REPRESENTATION

From (7) and (9) we note that a plot of log $(\rho_3 t^{3/2})$ versus t^{-1} yields a straight line. The observed intensity has been plotted in this manner for several solar flare events[1,2] and an appropriate straight line does indeed emerge for some events, over an interval that is approximately three times longer than the time required to achieve the peak intensity. We shall refer to this temporal domain as the "onset phase."

Deviations from isotropy are here investigated via a hybrid distribution function (f) that is a linear superposition of ρ_3 and ρ_1, viz

$$f(r,t) \equiv \frac{\rho_3(r,t)}{\rho_3(r,t_m)} + \epsilon \frac{\rho_1(r,t)}{\rho_1(r,t_m)} \tag{17}$$

where ϵ is a parameter that measures the amount of a one-dimensional contamination.

We shall find that a plot of log $(f\, t^{3/2})$ versus t^{-1} yields an approximately straight line during the onset phase for a variety of extreme situations. The cases to be considered are:

a) $\epsilon = 0 \quad D_3 = \frac{1}{3} v \lambda$

b) $\epsilon = 1 \quad D_1 = D_3 = \frac{1}{3} v \lambda$

c) $\epsilon = 1 \quad L_1 = L_3 = \lambda.$

The rôle of λ is exhibited by defining dimensionless variables,[12] as follows:

$$T \equiv (vt)/\lambda \tag{18}$$

$$S \equiv r/\lambda. \tag{19}$$

With this notation (18, 19), equation (9) may be re-written as

$$\frac{\rho_3(S,T)}{\rho_3(S,T_m)} = \left(\frac{2T}{S^2}\right)^{-3/2} \exp\left(\frac{-3[S^2 - 2T]}{4T}\right) \tag{20}$$

where

$$T_m = \frac{1}{2} S^2. \tag{21}$$

For the case (b) where $D_1 = D_3$, this implies $L_1 = \lambda/3$ and (10) is written

$$\frac{\rho_1'(S,T)}{\rho_1'(S,T_m')} = \left(\frac{2T}{3S^2}\right)^{-1/2} \exp\left(\frac{-[3S^2-2T]}{4T}\right) \tag{22}$$

where

$$T_m' = \frac{3}{2} S^2. \tag{23}$$

For the case (c) where $L_1 = \lambda$, (10) may be written

$$\frac{\rho_1(S,T)}{\rho_1(S,T_m)} = \left(\frac{2T}{S^2}\right)^{-1/2} \exp\left(\frac{-[S^2-2T]}{4T}\right) . \tag{24}$$

We note that $f(S,T)$ reaches a maximum, for cases (a) and (c), at $T = T_m$, given by (21), and that (22) is an extraordinary term in that it reaches a maximum at

$$T_m' = 3\, T_m. \tag{25}$$

For observations at the orbit of Earth, we have that

$$\lambda = 0.04 \text{ a.u.} \Rightarrow S = 25. \tag{26}$$

In the accompanying figure, we plot $\log\,[f\,T^{3/2}]$ as a function of T^{-1}, for $S = 25$, over an onset phase defined by the interval

$$(10^3 \approx 3T_m) > T > (10^2 = O[10S]). \tag{27}$$

Note that (25) and (27) imply

$$T < T_m'. \tag{28}$$

Therefore, the contamination of a one-dimensional diffusion component is liable to escape detection, under the circumstances of our present observations.

It is instructive to define an effective slope (θ) as

$$\theta \equiv \frac{\Delta[\log(f\ T^{3/2})]}{\Delta(T^{-1})} \tag{29}$$

for

$$\Delta(T^{-1}) \equiv (T_2)^{-1} - (T_1)^{-1} \tag{30}$$

where $T_2 = 10^3$ and $T_1 = 10^2$.

That the curves for cases (a), (b), and (c) are quantitatively similar is summarized by

$$\theta_c/\theta_b/\theta_a = 0.96/1.07/1.00 \tag{31}$$

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REFERENCES

1. D.A. Bryant, T.L. Cline, U.D. Desai, F.B. McDonald, Journal of Geophysical Research 67, 4983 (1962).

2. W. R. Webber, A Review of Solar Cosmic Ray Events, Section 27 of AAS-NASA Symposium On The Physics Of Solar Flares, edited by Wilmot N. Hess, NASA, SP-50 (1964).

3. E.N. Parker, The Passage of Energetic Charged Particles Through Interplanetary Space, University of Chicago EFINS-64-35 (1964).

4. N.F. Ness, C.S. Scearce, J.B. Seek, Initial Results of The IMP-1 Magnetic Field Experiment, NASA-Goddard X-612-64-95, April 1964.

5. G. Van Biesbroeck, Coronal Photographs, Chapter 9.3 of "The Sun," edited by Gerard P. Kuiper, University of Chicago Press (1953).

6. E.N. Parker, Astrophysical Journal 128, 664 (1958).

7. N.F. Ness and J.M. Wilcox, Physical Review Letters 13, 461 (1964).

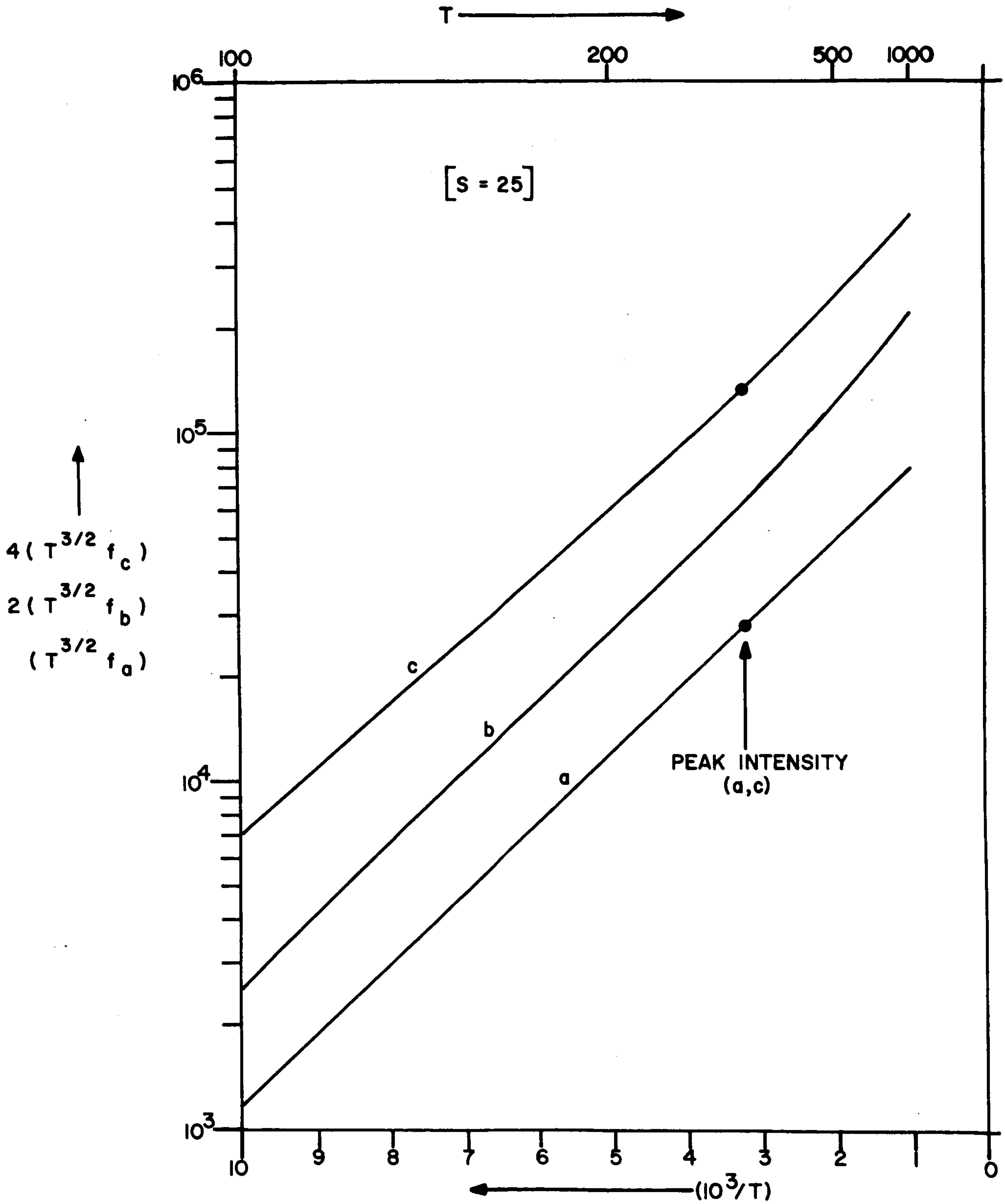
8. G. Klein, Proceedings of the Royal Society 211-A, 431 (1952).

9. E.N. Parker, Interplanetary Dynamical Processes, Interscience Publishers, New York (1963) Appendix B.

10. E.N. Parker, Interplanetary Dynamical Processes, Interscience Publishers, New York (1963) Chapter XIII-B-2.

11. S. Chandrasekhar, Reviews of Modern Physics 15, 1 (1943).

12. D.A. Bryant, T.L. Cline, U.D. Desai, and F.B. McDonald (to be published) Astrophysical Journal, February 15, 1965. These authors exhibit the utility of the variable (vt) in tracing the evolution of several solar proton events. For the event of 28 September 1961, the intensities of the various energy components (2.2 Mev → 430 Mev) are recorded as a function of (vt), and the intensity ratio of two such components appears essentially constant over a range of (vt) that extends from 2 a.u. to more than 100 a.u.



T
100
200
500
1000
10^6
10^5
10^4
10^3
[S = 25]
$4(T^{3/2} f_c)$
$2(T^{3/2} f_b)$
$(T^{3/2} f_a)$
c
b
a
PEAK INTENSITY
(a,c)
10
9
8
7
6
5
4
3
2
1
0
$(10^3/T)$